MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EMT3066 – RANDOM SIGNAL AND NETWORK ANALYSIS

(TE)

13 OCTOBER 2015
9:00 a.m. – 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **9 pages** with **4 questions** only. Attempt **ALL** questions. The distribution of the marks for each question is given.

2. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

3. Some common trigonometric identities, Fourier transform pairs, Fourier transform properties and Erlang-B table are supplied in the **Appendix**.

Question 1

(a) X_n is a zero-mean discrete-time random process and $k[n]$ is a deterministic sequence of n. Given that

$$Z_n = k[n] + X_n \ ,$$

i) provide expressions for the mean and variance of Z_n. [6 marks]

ii) find the cross-correlation and cross-covariance for X_n and Z_n. [4 marks]

iii) determine whether X_n and Z_n are uncorrelated, independent, or orthogonal random processes. [5 marks]

(b) Let $X(t)$ be a wide-sense stationary random process,

$$Z(t) = X(t+s) - \alpha X(t) \ ,$$

where α and s are constants.

i) determine whether $Z(t)$ is also a wide-sense stationary random process. [4 marks]

ii) find the cross covariance function of $X(t)$ and $Z(t)$, then determine whether these processes are jointly wide-sense stationary. [6 marks]

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Question 2

Let $X(t)$ and $Y(t)$ be jointly stationary WSS random processes.

(a) Given that $W(t) = X(t) - Y(t)$,

i) find $S_W(f)$ the power spectrum of $W(t)$, in terms of the power spectrum of $X(t)$ and $Y(t)$. [5 marks]

ii) if $Y(t)$ is the output of a filter $h(t)$, where $Y(t) = h(t) * X(t)$, re-express $S_W(f)$ in terms of $S_X(f)$ and the filter's transfer function only. [5 marks]

iii) write the integral to evaluate $E[W^2(t)]$. [2 marks]

(b) A zero-mean white noise $X(t)$ with power density $\frac{N_0}{2}$ is the input to a filter to produce output $Y(t)$. If the transfer function of the filter is given as

$$H(f) = \frac{1}{1 + j2\pi f},$$

i) find the cross power spectrum $S_{YX}(f)$ and the cross correlation $R_{YX}(\tau)$. [6 marks]

ii) find the output power spectrum $S_Y(f)$ and the output correlation function $R_Y(\tau)$. [6 marks]

iii) find the average power of the output. [2 marks]

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Question 3

A signal

$$x(t) = u(t)t^2 e^{-Kt},$$

was added to noise with power spectrum of $P / (K_N{}^2 + \omega^2)$, where K, P and K_N are positive constants. The sum is then applied to a matched filter.

a) Find the matched filter's transfer function. [7 marks]

b) Find the filter's impulse response. [7 marks]

c) Find the maximum signal to noise ratio at the output. [6 marks]

(Hint: $\int_{-\infty}^{\infty} \frac{dx}{(a^2+x^2)^3} = \frac{3}{8a^5}\pi$, $\int_{-\infty}^{\infty} \frac{x^2}{(a^2+x^2)^3} dx = \frac{\pi}{8a^3}$)

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Question 4

(a) At the local JPJ office, customers arrive according to a Poisson process at the rate of 12 customers per hour. There are two clerks to service the customers. Each clerk takes an exponentially distributed amount of time with mean of 5 minutes to service each customer.

i) Find the probability that an arriving customer must wait to be served. [5 marks]

ii) Find the mean number of customers in the office and the mean time spent in the office. [4 marks]

iii) Find the probability that there are 4 or more customers in the office. [4 marks]

(b) A particular department of a mobile phone manufacturer handles to types of jobs. Type 1 job requires the worker to scan the serial number of each item, this requires a constant service time of 1s. Type 2 job requires the worker to visually check the quality of main board assembly, which requires an exponentially distributed amount of time with mean 10s. Assume that the arrival rates of the two types of jobs are Poisson with the same rate.

Find the mean waiting time of the job if the department operates as follows:

i) An ordinary M/G/1 system [8 marks]

ii) A two priority M/G/1 system with priority given to type 1 jobs. [9 marks]

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APPENDIX A: Common Trigonometric Identities

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin(x)$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$2\cos(x) = e^{jx} + e^{-jx}$$

$$2j\sin(x) = e^{jx} - e^{-jx}$$

$$2\cos(x)\cos(y) = \cos(x - y) + \cos(x + y)$$

$$2\sin(x)\sin(y) = \cos(x - y) - \cos(x + y)$$

$$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y)$$

$$2\cos^2(x) = 1 + \cos(2x)$$

$$2\sin^2(x) = 1 - \cos(2x)$$

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APPENDIX B: Common Fourier Transform Pairs

Time	Frequency
$x(t)$	$X(f)$
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
sgn (t)	$\frac{1}{j\pi f}$
$\exp(j2\pi f_0 t)$	$\delta(f-f_0)$
$\exp(-\alpha t)u(t)$	$\frac{1}{\alpha+j2\pi f}$
$\exp(-\alpha \lvert t \rvert)$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$t\exp(-\alpha t)u(t)$,	$\left(\frac{1}{\alpha+j2\pi f}\right)^2$,
$\frac{t^{n-1}}{(n-1)!}\exp(-\alpha t)u(t)$	$\left(\frac{1}{\alpha+j2\pi f}\right)^n$
$\exp(-\alpha^2 t^2)$	$\sqrt{\pi/\alpha^2}\exp(-\pi^2 f^2/\alpha^2)$
$\cos 2\pi f_0 t$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$
$\sin 2\pi f_0 t$	$\frac{1}{2j}[\delta(f-f_0)-\delta(f+f_0)]$
$A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$	$A\tau\,\mathrm{sinc}(f\tau)$

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APPENDIX C: Fourier Transform Properties

Operation	Time	Frequency
Addition	$x_1(t)+x_2(t)$	$X_1(f)+X_2(f)$
Scalar multiplication	$kx(t)$	$kX(f)$
Duality/ Symmetry	$X(t)$	$x(-f)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shift	$x(t-t_o)$	$X(f)\exp(-j2\pi ft_o)$
Frequency shift	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Time convolution	$x_1(t)*x_2(t)$	$X_1(f)X_2(f)$
Frequency convolution	$x_1(t)x_2(t)$	$X_1(f)*X_2(f)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$
Conjugation	$x^*(t)$ $x^*(-t)$	$X^*(-f)$ $X^*(f)$

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APPENDIX D: Erlang-B Table

	Offered Load, ρ in Erl									
No of	Blockage Levels (%), $B(m,\rho)$									
servers, m	0.01	0.05	0.1	0.5	1	2	5	10	15	20
1	0.00	0.00	0.00	0.01	0.01	0.02	0.05	0.11	0.18	0.25
2	0.01	0.03	0.05	0.10	0.15	0.22	0.38	0.60	0.80	1.00
3	0.09	0.15	0.19	0.35	0.45	0.60	0.89	1.27	1.60	1.93
4	0.23	0.36	0.44	0.70	0.87	1.09	1.53	2.04	2.49	2.95
5	0.45	0.65	0.76	1.14	1.36	1.65	2.22	2.88	3.46	4.01
6	0.73	1.00	1.14	1.62	1.91	2.28	2.96	3.76	4.45	5.11
7	1.05	1.39	1.58	2.16	2.51	2.94	3.74	4.67	5.46	6.23
8	1.42	1.83	2.05	2.73	3.13	3.62	4.55	5.60	6.49	7.38
9	1.83	2.30	2.55	3.34	3.78	4.35	5.37	6.54	7.55	8.53
10	2.26	2.80	3.09	3.96	4.46	5.08	6.22	7.51	8.61	9.69
11	2.72	3.33	3.65	4.61	5.16	5.84	7.07	8.49	9.69	10.86
12	3.21	3.88	4.23	5.28	5.88	6.61	7.95	9.48	10.78	12.04
13	3.71	4.44	4.83	5.96	6.61	7.40	8.83	10.47	11.87	13.23
14	4.24	5.03	5.45	6.66	7.35	8.20	9.73	11.48	12.96	14.41
15	4.78	5.63	6.08	7.38	8.11	9.01	10.63	12.49	14.07	15.61
16	5.34	6.25	6.72	8.10	8.88	9.83	11.55	13.50	15.18	16.80
17	5.91	6.87	7.38	8.83	9.66	10.65	12.46	14.52	16.29	18.01
18	6.49	7.51	8.05	9.58	10.43	11.49	13.39	15.54	17.40	19.21
19	7.09	8.17	8.73	10.33	11.23	12.34	14.32	16.58	18.53	20.43
20	7.70	8.83	9.41	11.10	12.03	13.18	15.25	17.62	19.65	21.63
21	8.32	9.50	10.11	11.86	12.84	14.04	16.19	18.66	20.78	22.85
22	8.95	10.19	10.81	12.63	13.65	14.90	17.14	19.69	21.91	24.06
23	9.58	10.87	11.52	13.42	14.46	15.77	18.08	20.73	23.04	25.28
24	10.23	11.57	12.24	14.20	15.29	16.63	19.03	21.79	24.16	26.50
25	10.88	12.27	12.97	14.99	16.12	17.50	19.99	22.83	25.29	27.71
26	11.54	12.98	13.70	15.80	16.96	18.39	20.95	23.89	26.44	28.94
27	12.21	13.69	14.43	16.60	17.80	19.27	21.91	24.94	27.56	30.16
28	12.88	14.41	15.19	17.41	18.64	20.15	22.86	26.00	28.71	31.39
29	13.56	15.14	15.93	18.22	19.48	21.04	23.83	27.06	29.85	32.61
30	14.24	15.87	16.69	19.04	20.33	21.93	24.80	28.11	30.99	33.84
31	14.94	16.60	17.44	19.85	21.19	22.83	25.77	29.18	32.14	35.06
32	15.63	17.34	18.21	20.67	22.05	23.72	26.75	30.23	33.28	36.29
33	16.33	18.09	18.97	21.51	22.91	24.62	27.72	31.30	34.42	37.51
34	17.04	18.84	19.74	22.33	23.78	25.53	28.69	32.37	35.58	38.75
35	17.75	19.59	20.52	23.17	24.64	26.44	29.67	33.43	36.73	39.98
36	18.47	20.35	21.29	24.01	25.51	27.35	30.65	34.50	37.87	41.20
37	19.19	21.11	22.08	24.84	26.38	28.26	31.64	35.58	39.02	42.44
38	19.91	21.87	22.86	25.69	27.25	29.16	32.62	36.64	40.18	43.66
39	20.64	22.64	23.65	26.53	28.13	30.08	33.61	37.71	41.32	44.91
40	21.37	23.41	24.44	27.38	29.01	31.00	34.59	38.79	42.47	46.15

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